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# Malagasy *Sikidy:* A Case in Ethnomathematics

Marcia Ascher

*Department of Mathematics, Ithaca College, Ithaca, New York 14850*

*Sikidy* is a system of divination that plays a significant role in the lives of the people of Madagascar. Here we focus on the mathematical ideas which it embodies. Formal algebraic algorithms are applied to initial random data, and knowledge of the internal logic of the resulting array enables the diviner to check for and detect errors. *Sikidy* and the mathematical ideas within it are placed in their cultural and historical contexts. © 1997 Academic Press

*Sikidy* est un système de divination qui joue un rôle important dans la vie des Malgaches. Notre article porte sur les idées mathématiques que ce système renferme. Des algorithmes algébriques formels sont appliqués à des données initiales recueillies au hasard et la connaissance de la logique interne du tableau qui en résulte permet au devin de chercher et de détecter les erreurs. *Sikidy* et les idées mathématiques qu'il contient sont placés dans leurs contextes culturel et historique. © 1997 Academic Press

*Sikidy* ist ein Weissagungssystem, das eine bedeutende Rolle im Leben der Einwohner von Madagaskar spielt. Hier untersuchen wir die mathematischen Ideen, die es enthält. Formale algebraische Algorithmen werden auf zunächst wahllose Daten angewandt. Die Kenntnis der inneren Logik der daraus entstehenden Muster ermöglicht es dem Weissager, Fehler zu suchen und zu finden. *Sikidy* und die mathematischen Ideen, die darin enthalten sind, werden in ihrem kulturellen und historischen Kontext behandelt. © 1997 Academic Press

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## INTRODUCTION

Ethnomathematics involves the study of the mathematical ideas of people in traditional or small-scale cultures. Among mathematical ideas, we include those involving number, logic, and spatial configuration and, most important, the combination or organization of these ideas into systems or structures [1]. Here we examine the mathematical ideas embedded in a form of divination practiced in Madagascar. Our discussion places these ideas within their cultural, ideational, and historical contexts. To make the ideas comprehensible and to link them to some similar ideas elsewhere in the world, we draw upon what is familiar to us while taking care to distinguish our ideas and modes of description from the ideas of the Malagasy. By *our* ideas and modes of description, I mean those now shared by people schooled in the modern mathematical tradition and in digital computers.

Divination, in some form or another, has at some time been practiced by almost every known culture [21, 1]. In general, it is a decision-making process utilizing, as part of the process, a randomizing mechanism. The decisions sometimes involve the determination of the cause of some event, or more often, how, when, or whether



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to carry out some future action. In different cultures, and at different times, the randomizing mechanisms have varied considerably, involving animals or animal parts, lots or dice, sticks, or whatever can generate a set of different outcomes. The outcomes or results derived from them are then read and interpreted by the diviner. The divining practices and their cultural embedding differ widely, but what is most important is that within their cultural contexts, any particular form of divination is a shared, systematic approach to knowledge that is not otherwise attainable. Divination systems are, in fact, considered by some scholars to be sciences [22].

In the mathematics literature, a few forms of divination have received some attention. One of these is astragalomancy as practiced in classical Greece. In it, the numbers 1, 2, 3, 4 were associated with the four sides of an astragalus (an ankle bone of a hoofed animal). The sides that faced upward when a set of five astralagi were thrown were identified with a set of numbers, a god, and a prophecy. This form of divination has been linked to games of chance, the use of dice, and implicit and then explicit ideas about probability [6]. The Chinese use of the *I Ching* (Book of Changes) is another divination method that has received attention [12; 2, 112–113]. Beginning with a set of 49 sticks, an elaborate procedure is followed which includes three passes of randomly dividing the sticks into two piles, deleting groupings of four, and combining the remainder so that a set of six, seven, eight, or nine sticks results. If the result is even, it is represented by a broken line, and if it is odd, by a complete line. Going through the process six times generates a hexagram—that is, an ordered arrangement of six lines, each of which is either broken or unbroken. The hexagram directs the diviner to a portion of text to be applied to the particular question. Another hexagram derived from the first (lines generated by sixes or nines are changed to their opposite) indicates another text portion that refers to the future. An 11th-century ordering of the 64 possible hexagrams has sometimes been viewed as the first statement of binary representation; if 0 is associated with each unbroken line, and 1 with each broken line, the hexagrams, when interpreted as binary numbers, were in consecutive order from 0 to 63.

*Sikidy*, the particular system of divination that is the focus of this study, has a long and broad history. Its practice is ongoing and of great significance in Madagascar. Between the initial randomly generated data and the interpretation of the diviner, there is a formal algebraic algorithm that must be followed. What is more, the knowledge and concerns of the *ombiasy*—the diviners who are experts in *sikidy*—extend to some aspects of the logical structures involved. The mathematical ideas embedded in the practice of *sikidy* are richer, deeper, and more explicit than the two limited examples cited above; they fall primarily within what is categorized in modern mathematics as symbolic logic and include the computational concept of parity checking.

## CULTURAL CONTEXT

Madagascar is an island about 380 km off the east coast of Africa just opposite Mozambique. One of the largest islands in the world, it is approximately 1500 km long and 500 km wide and, in 1995, had a population of about 14 million people. Madagascar has about 20 different ethnic groups, but all of them share essentially the same language and much in the way of culture. The language is classed as a western Indonesian subgroup of the Malayo-Polynesian language family, and so the earliest immigrants are thought to be originally from the Malayan–Indonesian archipelago. The numeral words, in particular, are considered almost identical to some others in that language family, and some groups on the island have outrigger canoes, quadrilateral houses on stilts, and circular fishing nets, all usually associated with Indonesian and Malayo-Polynesian culture.

Although its mechanism is unclear, there was, in addition, significant Islamic influence from about 750 C.E. to 1150 C.E. While the religion itself was not adopted, traces of Arabic influence remain in the language, in particular in the names of lunar months and of days, and in the use of Arabic script. But, what is more, along with these, Arabic divination practices spread throughout Madagascar and were integrated with Malagasy practices. At about the same time, there began substantial and continuous importation of African slaves and, hence, aspects of African cultures. This diversity of people and cultures was fused and politically organized through an expansionist, feudal-type monarchy established by the indigenous Sakalava people in the 16th century and then superceded in the early 19th century by the indigenous Merina kingdom. In 1896, the island was colonized by the French. Then, in 1959, Madagascar became the independent Malagasy Republic.

Throughout all the political and cultural changes of the past four centuries and despite internal political and cultural differences, some form of divination has remained in every part of Madagascar. There are variations, but there are basic essential similarities. And, these similarities are intertwined with other shared aspects of the culture in which the divination is embedded, such as beliefs and practices related to ancestors and family tombs, residence and inheritance rules, and witchcraft. Because of the similarities, and to maintain a focus on mathematical ideas, we will, in our discussion of Malagasy divination, refer to the culture of Madagascar without distinguishing ethnic groups [14; 17; 24].

The *ombiasy* is a specialist in guiding people. He has a long apprenticeship, a formal initiation, knowledge of formal divining practices, and, above all, an interactive approach in which the use and interpretation of the divining materials are combined with asking the client questions and then phrasing new questions to guide the next stage in the divination. The interpretations are discussed with the client until the client determines the specific actions or answers that are relevant to solving his/her problems. The introduction and formal manipulations of the divining materials add a dimension that, while unfamiliar or odd to some of us, provides another mode of cognition as a focus for discussion [20].

The important place of divination and the diviner in Malagasy culture can be seen from the broad array of questions brought for resolution. Some questions involve the day on which something should be undertaken, whether it be a trip, planting, or ceremonial moving of the family tomb. There is a long tradition of adoption and fosterage, that is, placing children temporarily or permanently with other families. Hence, upon the birth of a child, the diviner is consulted to see how well the destinies of the child and its parents match or whether another family is preferable.

Other significant problems for resolution by divination are finding a spouse, finding lost objects, identifying thieves, and identifying the causes of illness, sterility, or any other misfortune. In Western medicine, for example, a virus may be considered the ''cause'' of an illness, but that does not answer the question of how, specifically, the illness was acquired, and why it was acquired by *that* individual at *that* time and in *that* place. To answer questions of cause, the Malagasy delve deeply, and the answers may well involve the actions of ancestors and/or witchcraft. Based on their knowledge and experience, some *ombiasy* are considered specialists and concentrate on dealing only with questions in their area of expertise.[1]

## MATHEMATICAL CONTEXT

The mathematical ideas involved in the practice of *sikidy* fall within what modern mathematicians categorize as symbolic logic and also include concepts of parity and parity checking. So, before turning to the details of *sikidy*, we draw together some of our related mathematical ideas.

In 1937, in his master's thesis at MIT, Claude E. Shannon introduced the use of symbolic logic to simplify switching circuits. One of his examples was the automatic addition of base 2 numbers using only relays and switches. The application was new and of extreme importance as base 2 representation and its automatic electrical manipulation became the internal operating mode for digital computers. The algebra of logic that he used, however, was about 100 years old, having been developed primarily by George Boole and discussed in his 1847 *The Mathematical Analysis of Logic.* One of Boole's stated concerns was to express ''logical propositions by symbols, the laws of whose combinations should be founded upon the laws of the mental processes which they represent . . .'' [4, 1857]. Further, the symbols need not be interpreted as magnitudes but ''every system of interpretation . . . is equally admissible, and it is thus that the same process may, under one scheme of interpretation, represent the solution of a question on the properties of numbers, under another, that of a geometrical problem, and under a third, that of a problem of dynamics or optics'' [4, 1856]. Or, as will be added here, the interpretation may represent the solution of a problem in divination.

In Boole's system there are, first of all, symbols which represent classes of objects, then rules of operation on the symbols, and, finally the observation that the rules

[1] Several different words are used to refer to the diviners and different authors translate the words somewhat differently. Block [3, 292], for example, distinguishes diviners (*mpiskidy*) and curers (*ombiasy*) and astrologers (*mpanandro*) but notes that they are often combined into one person; Decary [7, 5] uses the word *mpiskidy* as diviners but includes curing in their activities; Sussman and Sussman [25, 273] use *ombiasy* as a diviner–healer; and Rabedimy [23, 227] uses *ombiasy* as a diviner–healer who practices *sikidy* and who can also be an astrologer. We use *ombiasy* and include healing in the work of the diviner.

he established are the same as would hold in the two-valued numerical algebra of 0 and 1. Boole's basic operations on classes $x$ and $y$ were forming a new class of things that are either $x$'s *or* $y$'s but not both (referred to here as OR and symbolized by $+$), and forming a new class by selecting things that are both $x$'s *and* $y$'s (referred to here as AND and symbolized by $\cdot$). In terms of 0 and 1, the results of these operations are:

| AND($\cdot$) | OR($+$) |
|---|---|
| $0 \cdot 0 = 0$ | $0 + 0 =$ no interpretation |
| $1 \cdot 1 = 1$ | $1 + 1 =$ no interpretation |
| $1 \cdot 0 = 0$ | $1 + 0 = 1$ |
| $0 \cdot 1 = 0$ | $0 + 1 = 1.$ |

Later, Jevons, in his book *Pure Logic* of 1864, modified the OR operation and, in what is *now* known as Boolean algebra, it is

| OR($+$) |
|---|
| $0 + 0 = 0$ |
| $1 + 1 = 1$ |
| $1 + 0 = 1$ |
| $0 + 1 = 1.$ |

Thus, in general, $x \cdot x = x$ and $x + x = x$ and both OR and AND are commutative and associative. Not-$x$, now more commonly known as $\bar{x}$ or the complement of $x$, takes on the following values: for $x = 0$, $\bar{x} = 1$, and for $x = 1$, $\bar{x} = 0$ [15; 18].

For Shannon, the basic operations were the placing of switches in parallel ($+$) or series ($\cdot$), and the two values 1 and 0 were identified with closed and open switches (hence the transmission or nontransmission of electrical pulses.) Thus, for two switches in series, both must be closed for the overall circuit to transmit a pulse ($1 \cdot 1 = 1$ while $0 \cdot 1 = 0$, $1 \cdot 0 = 0$, $0 \cdot 0 = 0$); for two switches in parallel, at least one must be closed for the overall circuit to transmit a pulse ($1 + 1 = 1$, $1 + 0 = 1$, $0 + 1 = 1$, while $0 + 0 = 0$); and an inverter changes a pulse to no pulse and vice versa ($\bar{1} = 0$, $\bar{0} = 1$) [8; 16, 759].

Utilizing these basic operations, other operations can be constructed. The operation of particular interest here because of its central role in *sikidy* is the *exclusive or* referred to in current computer texts as XOR, denoted by $\oplus$. The computer circuit for this operation has been of special interest for several reasons. First, it is the major constituent of a "half-adder"; that is, it is a basic part of the circuitry for the addition of two binary numbers. When doing addition of two binary numbers, the half-adder gives the bit by bit result, while the full-adder also incorporates the carry from the previous bit position. The results of XOR are

$$0 \oplus 0 = 0$$

$$1 \oplus 1 = 0$$

$$1 \oplus 0 = 1$$

$$0 \oplus 1 = 1$$

and the operation is commutative and associative. In the symbolism of Boolean algebra, $x \oplus y$ can be expressed as $\bar{x} \cdot y + x \cdot \bar{y}$ or in numerous other ways including $(x + y) \cdot (\overline{x \cdot y})$. That, of course, is the point of Shannon's work—once one Boolean expression for a circuit is known, others can be derived algebraically and, from them, one can be selected for actual physical construction.

The current importance of XOR also stems from its use in the detection of errors in the electronic transmission of binary information. In 1948, Richard Hamming developed the idea of verifying whether received binary numbers were correct by checking whether the total number of 1s in them was odd or even. (To ensure that each should be even, a code word was constructed and sent along with the data [16, 763].) The XOR easily handles this parity testing: combining any number of 0s and 1s consecutively via XOR gives a 1 if the number of 1s is odd and a 0 if the number is even [19, 67–71]. This mode of error checking is used in *sikidy*.

In describing *sikidy* we use the modern terminology just introduced. Clearly, the Malagasy diviners did not express themselves this way, but our purpose here is to highlight and make comprehensible their mathematical ideas. We recognize, however, that we are limited—in what we can see and in what we can express—to ideas somewhat analogous to our own.

In *sikidy*, the classes in the two-valued logic are odd and even; the symbols for them are o (one seed) and oo (two seeds); and the basic operation combining two or more of these symbols is XOR, that is, the rules of combination are

$$\begin{aligned} \text{o} \oplus \text{o} \quad &= \text{oo} \\ \text{oo} \oplus \text{oo} &= \text{oo} \\ \text{o} \oplus \text{oo} &= \text{o} \\ \text{oo} \oplus \text{o} \quad &= \text{o}. \end{aligned}$$

Although the *ombiasy* did not call the operation he applied XOR, he did use these rules of combination. We, therefore, will use our descriptor XOR when describing what he did. All of this begins with randomly selected items (seeds) followed by the generation of a tableau via a lengthy formal algorithm. Once the tableau is generated, the parity check is done on one of its columns. The *ombiasy* knows that a particular column must have an even number of seeds in it and stops the divination if this is not the case. This is a particularly interesting parity check as the result depends on the algorithm being used and is independent of the initial random data. Further checking is also done based on other structural knowledge.

$$
\begin{array}{cccccc}
C_4 & C_3 & C_2 & C_1 & \\
\downarrow & \downarrow & \downarrow & \downarrow & \\
\hline
c_{41} & c_{31} & c_{21} & c_{11} & \leftarrow C_5 \\
c_{42} & c_{32} & c_{22} & c_{12} & \leftarrow C_6 \\
c_{43} & c_{33} & c_{23} & c_{13} & \leftarrow C_7 \\
c_{44} & c_{34} & c_{24} & c_{14} & \leftarrow C_8 \\
\hline
\end{array}
$$

$C_i$ for $i = 1, 2, 3, 4$ contains $c_{ij}$ for $j = 1, 2, 3, 4$

$C_i$ for $i = 5, 6, 7, 8$ contains $c_{j,\, i-4}$ for $j = 1, 2, 3, 4$

FIG. 1. The *mother-sikidy*.

## SIKIDY—THE BASIC FORMAL PROCEDURE

As part of the initiation process of an *ombiasy*, the initiate must ceremonially gather for his subsequent use between 124 and 200 dried seeds of a fano tree (*Piptaenia chrysostachys).*[2] To begin a divination session, the *ombiasy*, using various incantations, awakens the seeds in his bag and the verbal powers within him. The incantations include the origin myth of *sikidy* which links it both to the return by walking on water of Arab ancestors who had intermarried with Malagasy but then left, and to the names of the days of the week.

A fistful of seeds is taken from the diviner's bag and randomly lumped into four piles. Each pile is reduced by deleting two seeds at a time until either one or two are left in the pile. The four remainders become the entries in the first column of the tableau. The entire process, beginning with another selection of a fistful of seeds from the bag, is then repeated three more times each of which results in another column placed *to the left* of the previous column formed. Thus, there are four randomly generated columns of four entries each. Each of the entries can be one seed (o) or two seeds (oo), and so, in all, there are $2^4 = 16$ different columns possible and $16^4 = 65536$ different arrays possible. This randomly generated initial set of data is called *renin-sikidy* (the *mother-sikidy*), and each column and each row has a particular referent in the divination.

[2] The set of seeds is called *misikidy*, while the particular subset used for the divination is called *sikidy* which, at the same time, refers to their manipulation and the practice of divination itself. Since there are other modes of divination, some people specify this particular mode as *sikidy alanana.* And, as with most transliterations, there are other spellings, in particular *sikily* or *tsikily*. My discussions of the basic formal procedure and the means of checking the tableaux are based primarily on the descriptions in Decary [7], Sussman and Sussman [25], and Rabedimy [23].

To state the rest of the algorithm succinctly, and then to prove some of the structural results known to the *ombiasy*, we will identify the elements of the *mother-sikidy* as shown in Fig. 1. (The symbolism introduced in Fig. 1 involving $C_i$ and $c_{ij}$ is, of course, mine, not theirs.) Eight additional columns are generated according to the following algorithm:

$$\begin{array}{lll}
C_9: & c_{9j} = c_{8j} \oplus c_{7j} & j = 1, 2, 3, 4; \\
C_{10}: & c_{10,j} = c_{6j} \oplus c_{5j} & j = 1, 2, 3, 4; \\
C_{11}: & c_{11,j} = c_{4j} \oplus c_{3j} & j = 1, 2, 3, 4; \\
C_{12}: & c_{12,j} = c_{2j} \oplus c_{1j} & j = 1, 2, 3, 4;
\end{array}$$

$$\begin{array}{lll}
C_{13}: & c_{13,j} = c_{9j} \oplus c_{10,j} & j = 1, 2, 3, 4; \\
C_{14}: & c_{14,j} = c_{11,j} \oplus c_{12,j} & j = 1, 2, 3, 4;
\end{array}$$

$$\begin{array}{lll}
C_{15}: & c_{15,j} = c_{13,j} \oplus c_{14,j} & j = 1, 2, 3, 4;
\end{array}$$

$$\begin{array}{lll}
C_{16}: & c_{16,j} = c_{15,j} \oplus c_{1j} & j = 1, 2, 3, 4.
\end{array}$$

The order of generation follows [7, 21–26]. As these, the *descendants*, are generated, they are placed below the *mother-sikidy* so that the final tableau is as shown in Fig. 2. The referents for the 16 $C_i$ are listed in Fig. 3. Using an arbitrary set of data for $C_1$ to $C_4$, Fig. 4 shows an example of a tableau.

## CHECKING THE TABLEAU

As noted earlier, the *ombiasy* uses his knowledge of the logical structure of the tableau to check its validity before proceeding further with the divination. He knows several relationships that the tableau must contain regardless of the initial random data. These relationships are particularly interesting because of their differences:

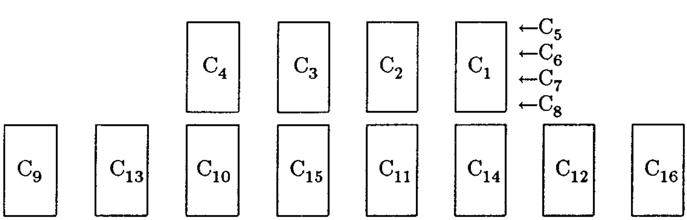



FIG. 2. The final tableau ($C_5$, $C_6$, $C_7$, $C_8$ are the rows of $C_1$, $C_2$, $C_3$, $C_4$).

$C_1$: the client (the person seeking the consultation)

$C_2$: material goods

$C_3$: a male evil-doer ( lit. -- the third)

$C_4$: the earth

$C_5$: the child

$C_6$: the bad intentions

$C_7$: a woman

$C_8$: the enemy (lit. -- the eight)

$C_9$: the spirit (lit. -- the ninth)

$C_{10}$: nourishment

$C_{11}$: the ancestors

$C_{12}$: the road

$C_{13}$: the diviner

$C_{14}$: the people

$C_{15}$: the creator

$C_{16}$: the house

FIG. 3. Column referents.

one involves examination of the overall tableau; another involves examining the results of combining some particular columns; and yet another, the parity check, involves examining only one specific column.

First of all, the *ombiasy* knows that at least two of the sixteen $C_i$ must be the same. In order to assure *ourselves* of this, consider that there are 16 $C_i$, and there are 16 different possible columns that can have one or two seeds in each of their four positions. So, if all 16 $C_i$ were different, they must contain one of each of the possible 16. Assuming that to be true, when the 16 $C_i$ are combined by XOR,

| 4 | 3 | 2 | 1 | |
|---|---|---|---|---|
| oo | o | oo | o | 5 |
| o | o | oo | oo | 6 |
| o | oo | o | o | 7 |
| oo | o | o | o | 8 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| oo | o | o | o | o | oo | o | oo |
| oo | oo | oo | oo | oo | oo | oo | oo |
| o | o | oo | oo | o | o | oo | o |
| o | oo | o | o | o | o | oo | oo |
| 9 | 13 | 10 | 15 | 11 | 14 | 12 | 16 |

FIG. 4. An example of a tableau.

position by position, the result must be $\begin{smallmatrix}\text{oo}\\ \text{oo}\\ \text{oo}\\ \text{oo}\end{smallmatrix}$ since there are eight possible columns with oo and eight possible columns with o in each of the four positions. Using the definitions of the $C_i$ and noting that $2(x) = x \oplus x = \text{oo}$ and $\text{oo} \oplus x = x$,

$$\underbrace{C_1 \oplus C_2}_{C_{12}} \oplus \underbrace{C_3 \oplus C_4}_{C_{11}} \oplus \underbrace{C_5 \oplus C_6}_{C_{10}} \oplus \underbrace{C_7 \oplus C_8}_{C_9} \oplus C_9 \oplus C_{10} \oplus C_{11} \oplus C_{12} \oplus \underbrace{C_{13} \oplus C_{14}}_{C_{15}}$$

$$\oplus\, C_{15} \oplus C_{16} = 2(C_9 \oplus C_{10} \oplus C_{11} \oplus C_{12} \oplus C_{15}) \oplus C_{16} = C_{16} = C_{15} \oplus C_1 = \begin{smallmatrix}\text{oo}\\ \text{oo}\\ \text{oo}\\ \text{oo}\end{smallmatrix}.$$

Hence, in each of the four positions, $C_{15}$ and $C_1$ are either both o or both oo, which contradicts the claim that the $C_i$ are all different. In our example (Fig. 4), due to the initial data, there are several repetitions within the tableau but, as we have shown, there must be some repetition.

The *ombiasy* call $C_{13}$ and $C_{16}$, $C_{14}$ and $C_1$, and $C_{11}$ and $C_2$ "the three inseparables" [23, 81]. Notice that by combining each pair via XOR, the results are the same. That is,

$$C_{13} \oplus C_{16} = C_{13} \oplus (C_{15} \oplus C_1) = C_{13} \oplus (C_{13} \oplus C_{14}) \oplus C_1 = C_{14} \oplus C_1 \quad \text{and}$$
$$C_{14} \oplus C_1 = (C_{11} \oplus C_{12}) \oplus C_1 = C_{11} \oplus (C_1 \oplus C_2) \oplus C_1 = C_{11} \oplus C_2.$$

In the example in Fig. 4, these "inseparables" all, of course, give the same result, namely, $\begin{smallmatrix}\text{oo}\\ \text{oo}\\ \text{oo}\\ \text{oo}\end{smallmatrix}$.[3]

Finally, we look at the parity check: the creator ($C_{15}$) must contain an even number of seeds. In the example in Fig. 4 it surely does. To prove that it always must, we show that when the four positions within $C_{15}$ are combined via XOR, the result must be oo. (We will adapt the summation symbol $\Sigma$ to $\bigoplus$ for combination via XOR.) Note that

$$C_{15} = C_{13} \oplus C_{14} = (C_9 \oplus C_{10}) \oplus (C_{11} \oplus C_{12}) = \bigoplus_{i=1}^{8} C_i.$$

Then, for each element within the column,

$$c_{15,j} = \bigoplus_{i=1}^{8} c_{ij} = \bigoplus_{i=1}^{4} c_{ij} \oplus \bigoplus_{i=5}^{8} c_{j,i-4}.$$

[3] There exists another set of "three inseparables" which are not mentioned as known to the Malagasy. They are $C_2$ and $C_{16}$, $C_{11}$ and $C_{13}$, and $C_{12}$ and $C_{15}$.

When the elements are combined,

$$\sum_{j=1}^{4} c_{15,j} = \sum_{j=1}^{4} \sum_{i=1}^{4} c_{ij} \oplus \sum_{j=1}^{4} \sum_{i=5}^{8} c_{j,i-4}$$

$$= \sum_{j=1}^{4} \sum_{i=1}^{4} c_{ij} \oplus \sum_{i=1}^{4} \sum_{j=1}^{4} c_{ij} = 2\left(\sum_{j=1}^{4} \sum_{i=1}^{4} c_{ij}\right) = \text{oo}.$$

Further, it can easily be demonstrated that $C_{15}$ is the only $C_i$ for which this is necessarily true.

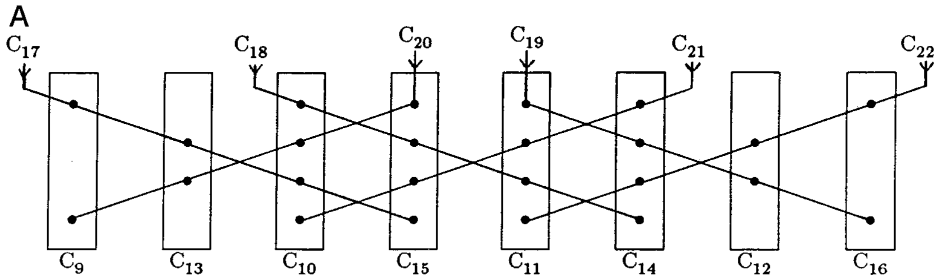
## PROCEEDING WITH THE DIVINATION

Once he has checked the tableau, the diviner's analysis begins. Many questions and answers rely on generating additional columns from the 16 $C_i$ so that for many divinations there are additional algorithms. These algorithms involve selections from the $C_i$ and, in some cases, generation of additional combinations with XOR [7, 77–79].

Although so far we have emphasized the logical algebra of the *sikidy* tableaux, some of these about 100 additional algorithms further direct our attention to the spatial interrelationships of the elements in the arrays. The algorithm that generated the *descendants* from the *mother-sikidy* first combined pairs of adjacent rows and then pairs of adjacent columns; those results were so positioned that the results of *their* subsequent pairwise combinations could be columns placed between the pairs. With the final combination of $C_{15}$ and $C_1$ into $C_{16}$, the *mother-sikidy* and its *descendants* formed the symmetric two-tiered array shown in Fig. 2. The additional algorithms introduce diagonal readings and more complicated, but still spatially patterned, orders of selections. For example, one secondary series, shown in Fig. 5A, is six columns ($C_i$; $i$ = 17, 18, ..., 22) selected from diagonal readings of the descendants and then three more ($C_{23}$, $C_{24}$, $C_{25}$) generated from these with XOR. Another example of a secondary series, shown in Fig. 5B, has four columns ($C_i$; $i$ = 26, 27, 28, 29) selected in another patterned way from the descendants and then two more ($C_{30}$, $C_{31}$) generated with XOR [6, 77–79].

In every case the result, of course, has to be one of the 16 possible columns made up of four entries each. The diviner's interpretation of the results and the generation and interpretation of additional results depend on which of the 16 appear and on their juxtaposition to each other. Interpretation is where the logical algebra leaves off and the attribution of meaning begins. The interpretations are not rote or standardized and vary with the ethnic group, the diviner, and, above all, the situation under discussion and the course of the discussion. There are, however, some shared themes that we explore further because of their additional mathematical ideas.

First of all, the diviners know the 16 possible outcomes and identify them by names, although the names may differ in different regions. Some, but not all, of the names are related to the names of the months. Also, for many diviners the 16 outcomes have particular directional associations. This imposition of spatial orientation echoes other aspects of Malagasy culture. In particular, there is strong




$$C_{17} = \begin{bmatrix} c_{9,\,1} \\ c_{13,\,2} \\ c_{10,\,3} \\ c_{15,\,4} \end{bmatrix} \qquad C_{18} = \begin{bmatrix} c_{10,\,1} \\ c_{15,\,2} \\ c_{11,\,3} \\ c_{14,\,4} \end{bmatrix} \qquad C_{19} = \begin{bmatrix} c_{11,\,1} \\ c_{14,\,2} \\ c_{12,\,3} \\ c_{16,\,4} \end{bmatrix}$$

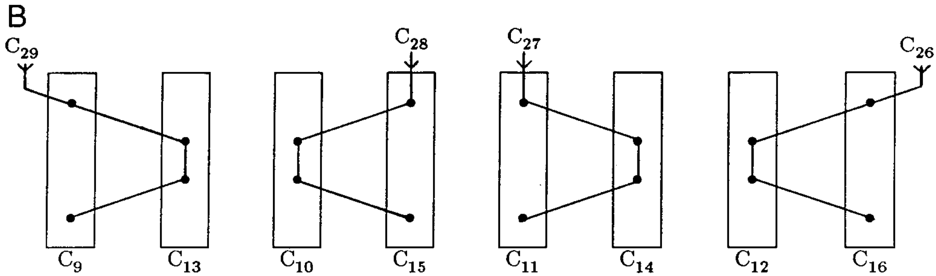
$$C_{20} = \begin{bmatrix} c_{15,\,1} \\ c_{10,\,2} \\ c_{13,\,3} \\ c_{9,\,4} \end{bmatrix} \qquad C_{21} = \begin{bmatrix} c_{14,\,1} \\ c_{11,\,2} \\ c_{15,\,3} \\ c_{10,\,4} \end{bmatrix} \qquad C_{22} = \begin{bmatrix} c_{16,\,1} \\ c_{12,\,2} \\ c_{14,\,3} \\ c_{11,\,4} \end{bmatrix}$$

$$C_{23} = C_{17} \oplus C_{20} \qquad C_{24} = C_{18} \oplus C_{21} \qquad C_{25} = C_{19} \oplus C_{22}$$

FIG. 5. (A) A secondary series utilizing diagonal readings.

belief in astrology and, in that, 12 month names are associated with 12 radial directions. Further, directionality is a significant part of daily life; different directions have different values—the northeast is good while the southwest is lacking in virtue, and the directions between vary in religious and moral value. As a result, there are prescribed house and tomb orientations and prescribed interior layouts extending to where specific items need to be stored and how visitors position themselves when gathered in a room. The positioning of visitors joins direction with social status—the most important stand toward the northeast and the least important toward the southwest [3, 288–291; 23, 111–113].

Similarly, in *sikidy*, the spatial associations of the 16 possible outcomes have social concomitants. The 8 outcomes that have an even number of seeds are designated as




$$C_{26} = \begin{bmatrix} c_{16,1} \\ c_{12,2} \\ c_{13,3} \\ c_{16,4} \end{bmatrix} \qquad C_{24} = \begin{bmatrix} c_{11,1} \\ c_{14,2} \\ c_{14,3} \\ c_{11,4} \end{bmatrix} \qquad C_{30} = C_{26} \oplus C_{27}$$

$$C_{28} = \begin{bmatrix} c_{15,1} \\ c_{10,2} \\ c_{10,3} \\ c_{15,4} \end{bmatrix} \qquad C_{29} = \begin{bmatrix} c_{9,1} \\ c_{13,2} \\ c_{13,3} \\ c_{9,4} \end{bmatrix} \qquad C_{31} = C_{28} \oplus C_{29}$$

FIG. 5. (B) Another secondary series.

princes and the 8 with an odd number of seeds are slaves. Each of the 16 possible outcomes has a specific place in a square whose sides are associated with the four cardinal directions. Their positions, as designated by the Malagasy, are shown in Fig. 6. The square is separated into halves by the diagonal that joins its northeastern and southwestern corners—the northwestern half is called the Land of Slaves and the southeastern half is called the Land of Princes. However, the Land of Slaves does not only contain slaves nor does the Land of Princes only contain princes. As can be seen in Fig. 6, there are slaves and princes in each half, but there are also two migrators, that is, one slave and one prince that move, more or less, with the sun, and so their place depends on the time of day of the divination. From sunup to 10 a.m. the migrators are in the east, from 10 a.m. to 3 p.m. in the north, from 3 p.m. to sunset in the west, and never in the south as divination does not take place at night. (Some diviners always associate the migrators with the west.) Power inequalities result from rank and place: princes are more powerful than slaves; slaves (or princes) from the Land of Princes are more powerful than slaves (or

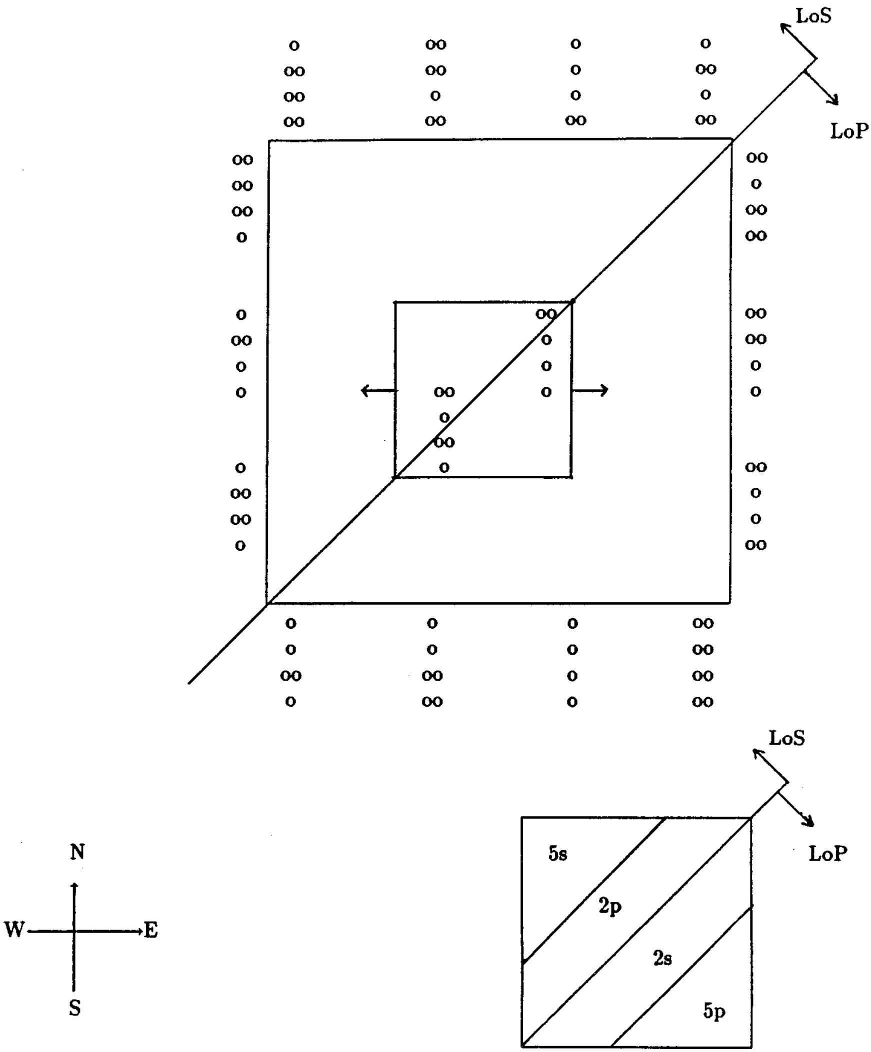



FIG. 6. Spatial associations of the 16 possible outcomes [7, 13]. The two in the center are the migrators. Note the symmetric placement of slaves and princes in their respective lands.

princes) from the Land of Slaves; slaves from the same land are never harmful to each other; and battles between two princes from the Land of Princes are always serious but never end in death [7, 11–20; 23, 114–116].

An example of a divination that makes use of these relationships is one related to illness. If, in the final tableau, the client ($C_1$) and the creator ($C_{15}$) are the same, there definitely will be recovery; if the client and the ancestors ($C_{11}$) are the same, the illness is due to some discontent on the part of the ancestors; and if the client and the house ($C_{16}$) are the same, the illness is the same as an earlier one from which there has been recovery. The result of the combination $C_1 \oplus C_9$ has the illness itself as a referent. If, for example, the client is a slave of the east and the illness is a prince of the south, the client is dominated by the illness and so it is serious. However, since both the east and south are in the Land of Princes, the illness will not lead to death. If, on the other hand, the client were a prince of the north (Land of Slaves), there would be a strong battle with a good chance that the ill person would die. Some tableaux are considered to be exceptionally serious and quite hopeless. The most extreme is "the red *sikidy*," in which $C_1$, $C_2$ and $C_3$, and $C_4$ are all $\begin{smallmatrix} \mathrm{oo} \\ \mathrm{oo} \\ \mathrm{oo} \\ \mathrm{oo} \end{smallmatrix}$. In this case, all 16 $C_i$ in the tableau are the same. Some of the initial questions and answers in a divination related to illness are straightforward but, in general, the divination will continue with less programmatic combinations to answer further questions on causes and cures [7, 27–30, 38].[4]

## SIKIDY-UNIQUE

The attribution of directionality to the outcomes gives rise to tableaux with special importance. The power to see into the past or future is greater if all four regions, east, west, north, and south, are represented in a tableau. Tableaux with the most power, however, are those in which all four regions are represented but at least one region has only one representative. These tableaux are referred to as *tokan-sikidy* (*sikidy-unique*), and they hold special abstract interest for the *ombiasy*. They are sought by the *ombiasy* for themselves; that is, in addition to simply encountering them in the course of divinatory consultations, finding beginning data which lead to such tableaux is an intellectual problem in and of itself. Knowing as many as possible leads to an increase in prestige: some such tableaux are publicized by being posted on doors; some are shared with other *ombiasy* by word of mouth; and there is speculation, but with no persuasive evidence, that some *ombiasy* have secret rules for generating certain types of *sikidy-unique* [23, 162]. No one knows all of them or how many there are, and so the search for them continues. Two examples of *sikidy-unique* are shown in Figs. 7 and 8. In Fig. 7, $C_8$ is the unique representative of the south. Note that $C_{10}$ is a migrator but that causes no problem as a migrator can only be in the east, west, or north and never the south. The tableau in Fig. 8 is unusual in having the creator ($C_{15}$) as a unique representative from the east, as well as having a unique representative from the north ($C_{16}$)—all the rest of the $C_i$ are from the south.

[4] Because of the programmatic nature of some of the initial questions and answers in a divination related to illness, a recent study utilized a computer to simulate the process [11]. The study compared the probabilities of various results. It found, for example, that, assuming a male client, the cause of illness was sorcery in 21.1% of the possible 65,536 tableaux, witchcraft for 16.5% of them, spirit possession for 9.6%, the chief for 2.6%, food contamination for .8%, ancestors for .7%, and undetermined for 48.7%. Beyond a few more questions, the researcher noted that the diviners' decision-making procedures were not amenable to computerization. It is important to note that although the divination begins with a random process, I have read nothing that indicated that the Malagasy view divination as related to chance.

| 4(E) | 3(E) | 2(W) | 1(E) | |
|---|---|---|---|---|
| oo | oo | o | oo | 5(E) |
| o | o | oo | oo | 6(E) |
| oo | o | oo | o | 7(N) |
| oo | oo | o | o | 8(S) |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| oo | oo | oo | o | oo | o | o | o |
| o | oo | o | oo | oo | oo | oo | oo |
| o | oo | o | oo | o | oo | o | o |
| oo | o | o | o | oo | oo | oo | oo |
| 9(E) | 13(W) | 10(Mig) | 15(W) | 11(N) | 14(N) | 12(N) | 16(N) |

FIG. 7. A *sikidy-unique* [7, 37]. $C_8$ is the sole representative of the south.

## HISTORICAL LINKAGES

Several of the mathematical ideas within *sikidy* are shared with or derived from early Arabic divination; nevertheless, at the same time, its deep embedding in Malagasy culture and its significance in the culture make it distinctively Malagasy.

We cannot know for sure how early *sikidy* was developed, but missionary reports of the early 1600s attest to the fact that *sikidy* was, by then, already quite well established. The question of when and how Arabic influence first reached Madagascar has received considerable attention but has not been, and probably cannot be, resolved. Although some put it earlier and others later, most writers link it to the

| 4(S) | 3(W) | 2(S) | 1(S) | |
|---|---|---|---|---|
| o | oo | o | o | 5(S) |
| o | oo | o | o | 6(S) |
| oo | oo | oo | oo | 7(S) |
| o | o | o | o | 8(S) |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| o | o | oo | oo | o | o | oo | o |
| o | o | oo | oo | o | o | oo | o |
| o | o | oo | o | oo | oo | oo | o |
| o | o | oo | o | oo | oo | oo | oo |
| 9(S) | 13(S) | 10(S) | 15(E) | 11(S) | 14(S) | 12(S) | 16(N) |

FIG. 8. A *sikidy-unique* [23, 159]. $C_{15}$, $C_3$, and $C_{16}$ are the sole representatives of the east, west, and north, respectively.

sea-going trade involving the southwest coast of India, the Persian Gulf, and the east coast of Africa in the 9th or 10th century C.E. However or whenever the interaction occurred, the Arabic connection is certain; not only is it reiterated in the *sikidy* origin myth, but Malagasy divination is intertwined with Islamic day and month names and Arabic script [3, 278–283; 10; 17, Ch. 3; 23, 17–28].

The Malagasy use of Arabic script is an unusual chapter in the history of literacy; the script was adapted to the writing of the Malagasy language in which the speech sounds differ considerably from Arabic. Scholars of Arabic, unfamiliar with the Malagasy language, are said to have referred to the modified script as "a kind of picture puzzle" [3, 283]. As with *sikidy*, by the 1600s there are reports of the presence of the modified script in Madagascar. But even more important for our study of *sikidy*, the subject of many of the writings are medicine, astrology, and divination, and many of the writings are considered to be translations or adaptations of Arabic manuscripts [3; 10].

The missionary report of 1616 notes that at that time in Madagascar there were two forms of *sikidy*; one utilized the seeds of a tree, and one utilized finger imprints made in the sand [23, 14–15]. The other missionary, writing in 1661, refers to *sikidy* as a variation of *geomancy* [10, 104]. In general, in the context of divination, *geomancy*, a word of Western origin, has several referents, none of which is originally Western. The word implies divination using directions or the earth. In the late 1800s, Paul Tannery, a French historian of mathematics, set himself to finding the first usage of the word as applied to the particular form of divination that interests us [26]. He concluded that the word's first use was in the first half of the 12th century in a Western Latin text translated from the Arabic by Hugo Sanccelliensis. Further, he concluded that the procedures described in the text, which were in vogue throughout Europe well into the 16th century, were not known in the West before they were introduced by the Arabs. Bernard Carra de Vaux [5] cites extensive references in Arabic writings to this mode of divination called by the Arabs *ilm er-raml* or *science of the sand.* He cites Zénâti of the Berber subgroup,[5] the Zénâtah, as a major author but further notes that students of Zénâti list earlier teachers of Zénâti extending back to a Berber contemporary of Mohammed and before him to Tomtom el-Hindi, and still earlier going back to Idrîs, the god of writing. In any case, according to Toufic Fahd [9], sand divination was spread by Arab scholars, probably in the 8th and 9th centuries, to Damascus, to Alexandria, to Cairo, into the Sudan, into Spain, and, later, into numerous other places including France and Germany. And, there are a number of places where some forms of it still exist. Further, it has been argued that some of the ongoing varieties in West Africa pre-dated and fed into the Arab version [13].

There is no way to know exactly where or by whom the idea was started. Clearly, it was shared by several peoples as they interacted and, in the process of sharing, it was modified, mixed with other ideas, and adapted to different cultures. *Sikidy*,

[5] The Berbers are the indigenous people of North Africa, and Berber is classed as a Hamitic (Afro-Asiatic) language.

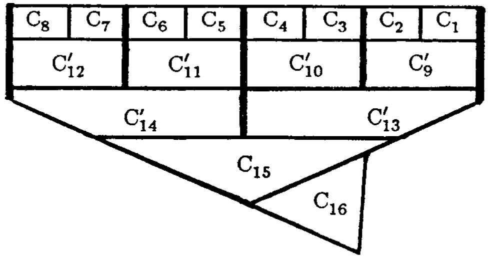



FIG. 9. The final tableau in geomancy. (For comparison to Fig. 2, the final tableau in *sikidy*: $C_1$–$C_8$ have the same values; $C'_9 = C_{12}$; $C'_{10} = C_{11}$; $C'_{11} = C_{10}$; $C'_{12} = C_9$; $C'_{13} = C_{14}$; $C'_{14} = C_{13}$; $C_{15}$–$C_{16}$ have the same values.)

then, is another branch of the tree-like history of *ilm er-raml*/geomancy and, hence, some of the mathematical ideas in it are shared with others. If we assume the writing of Hugo Sanccelliensis to be a transmission point between the Arab world and Europe, Tannery's summary of geomancy based on it [26, 344–350] provides insight into which ideas the Malagasy shared with the Arabs and which were then shared with the Europeans.

Primarily we see that the basic algorithm and tableau are indeed shared, although the tableau has a different spatial arrangement. In geomancy, as it is called by Sanccelliensis, the first four columns each have one or two dots in each of four positions. There is no specification as to how these are determined except that they are arrived at randomly. The next four columns are the transpose of the first four and the eight together are referred to as the *mothers*. The four generated by combining these in pairs via XOR (the same pairs as in *sikidy*) are the *daughters*, the four generated from them are the *granddaughters*, and the two derived from them are the *witnesses*. The next one, gotten by combining the witnesses via XOR, is referred to as the *judge* and the last, the sixteenth, is only completed if the first 15 are insufficient for the questions addressed in the divination. When the sixteenth is used, it is, as in *sikidy*, the XOR combination of the fifteenth and the first. As the columns are formed, they are placed in *houses* as shown in Fig. 9. It is also significant that the parity check, that is, the fact that the number of points in $C_{15}$ must be even, is also noted here, although the other checks, additional combining algorithms, and anything comparable to *sikidy-unique* are not.

We referred earlier to the modern context of the mathematical ideas under discussion including such terms as symbolic logic, two-valued logic, parity, and parity checking. In many cases in ethnomathematics, a traditional culture's expression of some mathematical idea is unique to that culture and has no bearing on the appearance of the ideas in other cultures [1]. Here that is clearly not the case. Since ideas are spread through human exchanges, we cannot know how or whether *sikidy*, in turn, influenced the mode of divination from which it was derived. But we do know with certainty that the Arabic *ilm er-raml*, with its basic algorithm involving a two-valued logic and parity check, became popular and widespread in Europe under the name of *geomancy.* Hence, the mathematical ideas shared by geomancy and *sikidy* were present in a number of cultures in the context of divination prior to what is generally viewed as their appearance in modern mathematics.

## CONCLUSION

The significant role of the *ombiasy* and of *sikidy* in Malagasy culture has persisted for several hundred years. As with other forms of divination, *sikidy* begins with a randomizing mechanism, but then it builds upon that with formal algebraic processes. As an integral part of guiding and interpreting the divination, the *ombiasy* knows a wide variety of algorithms and is knowledgeable about structural interrelationships of the resulting arrays. And the search for *sikidy-unique* further demonstrates the *ombiasy*'s concern for the logical structure of an array. Some of the interpretations relate outcomes with directionality, and concepts of oddness and evenness are basic to the entire process. In all, *sikidy* joins together a diversity of mathematical ideas to build a framework for decision-making that affects the lives of the Malagasy on a daily basis.

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