A fuzzy approach to membrane computing with approximate copies

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1 Introduction

Membrane computing is a formal computational paradigm, invented in 1998 by Gh. Păun [5], that rewrites multisets of objects within a spatial structure inspired by the membrane structure of living cells and according to evolution rules that are reminiscent of the processes that take place inside cells. In this paper we use techniques based on fuzzy sets to develop a general membrane computing model that takes into account the imperfection of the reactives involved in computations. I.e., the fact that the actual objects used in computations, as well as the actual output of the latter, need not be exact copies of the reactives that are assumed to be used in the computations or to be produced by them but only approximate copies. This is a generalization of our previous work [1]. Other fuzzy approaches to membrane computing have been proposed in [3, 4, 6].

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2 The model

Given an alphabet V, we denote by V^* the set of words over V. Given a word $\underline{u} \in V^*$, we denote by $|\underline{u}|$ the length of $\underline{u}$ and, given a letter $v \in V$, by $|\underline{u}|_v$ the number of occurrences of v in w.

A *fuzzy subset* of a set X is a mapping from X to the unit interval $I = [0, 1]$. Such a fuzzy subset *finite-valued* when its image is a finite subset of I. For every fuzzy subset $\varphi : X \to I$, its *t-cut*, for every $t \in [0, 1]$, is

$$\varphi_t = \{x \in X \mid \varphi(x) \geq t\}.$$

Roughly described, a fuzzy P-system will be a structure similar to a crisp P-system, supported on a membrane structure that defines regions whose contents evolve following rules that created, destroy and move reactives. Now, we shall use *reactives* as "ideal definitions" of chemical compounds, and hence they are fuzzy subsets of X: for every reactive $v : X \to [0,1]$, we understand that $v(x) = t$ denotes that the object $x \in X$ is a copy of $v \in V$ with a degree t of exactitude. So, $v(x) = 1$ means that x is an *exact copy* of the reactive v, and $v(x) = 0$ means that x cannot represent in any way the reactive v. We shall say that an object $x \in X$ is *similar* to a reactive $v \in V$ when $v(x) > 0$ and we shall assume in this paper that each object in X is similar to at most one reactive.

As in the crisp case, fuzzy P-systems will be supported by a membrane structure. Recall that a *membrane structure* μ is a finite rooted tree whose nodes are called *membranes*. We shall always denote by M the set of membranes of a membrane structure. This tree represents a hierarchical structure of nested membranes, with the edges representing the relation 'being directly inside.' The tree's root 1 is then called the *skin membrane*. We expand every membrane structure μ by adding a new node to it labelled *env* and an arc going from 1 to *env*; let $\overline{\mu}$ denote the resulting tree and $\overline{M}$ its set of nodes $M \cup \{env\}$. This new node *env* is called the *environment*, because it surrounds the skin membrane.

We understand that every $m \in \overline{M}$ defines a *region* K_m. At each moment, every such region contains a set of objects. Now, he reactives being fuzzy sets, the content of these regions at each moment will be formally described by means of an $\overline{M}$-indexed family of *fuzzy multisets* over a set V of reactives. These fuzzy multisets specify, for every $v \in V$ and for every value $t \in]0,1]$, how many objects in each region K_m are copies of the reactive v with degree of accuracy t. They are *ac-fuzzy multisets* in the sense of [2].

A *configuration* for this fuzzy P-system, with set of membranes M and set of reactives V, will be a family of fuzzy multisets $(F_m)_{m \in \overline{M}}$ over V,

$$F_m : V \times I^+ \to \mathbb{N}_\infty, \qquad m \in \overline{M}.$$

Each such mapping F_m specifies, for every $v \in V$ and for every $t \in I^+$, how many objects exist in the region K_m such that $v(x) = t$ in a configuration.

Now, *fuzzy P system* is a structure

$$\Pi = (V, \mu, m_{out}, (S_m)_{m \in M}, (\mathcal{R}_m)_{m \in M})$$

where:

- V is the finite set of *reactives* used by the membrane system.
- μ is a membrane structure, with set of membranes M.
- $m_{out} \in M$ is the *output membrane*.
- $(S_m)_{m \in \overline{M}}$ is a family of finite fuzzy multisets over V, called the *initial configuration*, which describes the initial content of all regions K_m.

- For every membrane $m \in M$, $\mathcal{R}_m$ is a finite set of *evolution rules* associated to the membrane m.

  Each evolution rule in $\mathcal{R}_m$ has the form

  $$R = ((\underline{c}; \underline{a} \to \underline{(b, m)}), \tau, \phi),$$

  where:

  — $\underline{c} \in V^*$ represents the *catalysts* necessary for the reaction represented by the rule to take place; the (possibly inexact copies of these) catalysts used to trigger the application of this rule are not modified in any way by this application.

  — $\underline{a} \in V^*$ represents the reactives that are processed by the reaction represented by the rule; we shall call them the *active reactives* of this rule. These reactives are *spent*, destroyed, when the rule is applied.

  For simplicity, we assume that, for every $v \in V$, if $|\underline{c}|_v > 0$, then $|\underline{a}|_v = 0$: i.e., any reactive that is a catalyst of a rule cannot be an active reactive of this rule —although it could be an active reactive for some other rule in the same region or in another one.

  — $\underline{(b, m)} \in (V \times (M \cup \{env\}))^*$ represents the reactives that are produced by the reaction represented by the rule, together with the region where each one of them is placed: every symbol (b', m') in this word means that a (possibly inexact copy of a) new reactive b' is produced in the region $C_{m'}$ when the reaction represented by this rule takes place.

  We assume that, for every symbol (b', m') appearing in the word $\underline{(b, m)}$, the membrane m' is adjacent to m in the expanded tree $(M, E)^{(env)}$, i.e., m' is directly included in m or m is directly included in m'. If $m' = env$, so that $m = 1$, the object represented by b' is moved to the environment, leaving the membrane system and never coming back (notice that no rule is defined in the environment).

  — $\tau : V \to [0, 1]$ is a *threshold* function that determines the degree of similarity to every reactive appearing in $\underline{c}$ or $\underline{a}$ necessary for an object to be considered as such a reactive to the effect of triggering an application of this rule.

  We impose on τ the condition that $\tau(v) > 0$ for every reactive that is a catalyst or an active reactive of this rule. On the other hand, for simplicity, we do not impose any threshold on the reactives that are not either catalysts or active in a given rule: i.e., we assume that, if $v \in V$ is such that $|\underline{c}|_v = 0$ and $|\underline{a}|_v = 0$, then $\tau(v) = 0$.

  — $\phi :]0, 1]^{|\underline{c}|} \times]0, 1]^{|\underline{a}|} \to]0, 1]$ is a function that determines the value of similarity of all objects produced by the reaction to the reactives supposed to be obtained (as specified by the word $\underline{(b, m)}$) in terms of the similarity of the actual objects used in it to the catalysts and active reactives of the rule.

Given a configuration $(F_m)_{m\in M}$,for every $m \in M$ we shall denote by $F_m[v]$ the set of those values $t \in]0, 1]$ such that $F_m(v, t) > 0$: these are non-zero degrees of exactitude of the copies of v that exist in C_m in the moment described by the configuration.

An evolution rule

$$R = (\underline{c}; \underline{a} \to \underline{(b, m)}, \tau, \phi)$$

in $\mathcal{R}_{m_0}$ *can be triggered* in a configuration $(F_m)_{m\in M}$ when, for every $v \in V$,

$$\sum_{t\geq\tau_{in}(v)} F_{\varepsilon(m_0)}(v, t) \geq |\underline{c} \cdot \underline{a}|_v.$$

This means that there are more copies in K_{m_0} within the degree of accuracy required by the threshold functions, than the specified quantities.

When a rule

$$R = (\underline{c}; \underline{a} \to \underline{(b, m)}, \phi, \tau)$$

in $\mathcal{R}_{m_0}$ can be triggered in a configuration $(F_m)_{m\in M}$, an *application* of it modifies this configuration into a new configuration $(F'_m)_{m\in M}$, which we call the *result* of this specific application. This new configuration is obtained in the following way. To simplify the notations, let

$$\begin{aligned} K(R) &= \{v \in V \mid |\underline{c}|_v > 0\} \\ A(R) &= \{v \in V \mid |\underline{a}|_v > 0\} \\ B_{m'}(R) &= \{v \in V \mid |\underline{(b, m)}|_{(v,m')} > 0\}, \quad m' \in M; \end{aligned}$$

recall that, by assumption, $K(R) \cap A(R) = \emptyset$. For every $v \in K(R) \cup A(R)$, let

$$\ell(v) = |\underline{c}|_v + |\underline{a}|_v$$

and for every $m' \in M$ and for every $v \in B_{m'}(R)$, let

$$r_m(v) = |\underline{(b, m)}|_{(v,m)}.$$

Now, $(F'_m)_{m\in M}$ is obtained by performing the following steps:

(1) For every $v \in K(R)\cup A(R)$, which are the reactives that are either catalysts or active for R, we choose $\ell(v)$ objects in C_{m_0} with degree of similarity with v at least $\tau(v)$. Formally, to do it, for every $v \in K(R) \cup A(R)$, if

$$F_m[v] \cap [\tau(v), 1] = \{t_{v,1}, \ldots, t_{v,h_v}\}, \quad \text{with } t_{v,1} < \cdots < t_{v,h_v},$$

then we form a vector

$$\iota(v) = (\overbrace{t_{v,1}, \ldots, t_{v,1}}^{p_{v,1}}, \ldots \overbrace{t_{v,h_v}, \ldots, t_{v,h_v}}^{p_{v,h_v}})$$

such that $0 \leq p_{v,j} \leq F_{m_0}(v, t_{v,j})$ for every $j = 1, \ldots, h_v$ and $\sum_{j=1}^{h_v} p_{v,j} = \ell(v)$.

This corresponds to choosing $p_{v,1}$ objects x such that $v(x) = t_{v,1}$, $p_{v,2}$ objects x such that $v(x) = t_{v,2}$, and so on, up to $\ell(v)$ objects. These objects (or rather, the number of objects within each degree of similarity with v) are chosen in a non-deterministic way: forming a different such vector would correspond to a different application of the rule and hence it could lead to a different result. Notice also that the actual objects are unrelevant, only their degree of similarity with v.

(2) We remove from C_{m_0} the chosen inexact copies of the active reactives of R. Formally, we define, for every $m \in M$, a mapping $\widetilde{F}_m : V \times]0,1] \to \mathbb{N}$ as follows: $\widetilde{F}_m = F_m$ for every $m \neq m_0$, and

- for every $v \in A(R)$,

$$\widetilde{F}_{m_0}(v, t_{v,j}) = F_{m_0}(v, t_{v,j}) - p_{v,j} \text{ for every } t_{v,j} \in F_m[v] \cap [\tau(v), 1]$$
$$\widetilde{F}_{m_0}(v, t) = F_{m_0}(v, t) \text{ if } t \notin F_m[v] \cap [\tau(v), 1]\}$$

- for every $v \notin A(R)$, $\widetilde{F}_{m_0}(v, t) = F_{m_0}(v, t)$ for every $t \in]0, 1]$.

Notice in particular that $\widetilde{F}_{m_0}(v, -)$ is not modified for any catalyst of the rule. This corresponds to the fact that catalysts of a reaction are not modified by the reaction: they only must be there for the reaction to be triggered.

(3) Let

$$t_{app} = \Phi((\iota(v))_{v \in K(R) \cup A(R)}) \in]0, 1].$$

Notice that t_{app} depends on the rule (the mapping F) as well as on how much the chosen objects were similar to the necessary reactives. Now, to every region C_m with m adjacent to m_0 and and for every $v \in B_m$, we add $r_m(v)$ copies x of v with degree of exactitude t_{app}; notice that, by assumption, these objects have degree of similarity 0 with any other reactive $v' \neq v$.

Formally, this defines, for every $m \in M$, a mapping $F'_m : V \times]0, 1] \to \mathbb{N}$ as follows:

- for every $m' \in M$ and for every $v \in B_{m'}(R)$,

$$F'_{m'}(v, t_{app}) = \widetilde{F}_{m'}(v, t_{app}) + r_{m'}(v)$$
$$F'_{m'}(v, t) = \widetilde{F}_{m'}(v, t) \quad \text{if } t \neq t_{app}$$

- for every $m' \in M$ and for every $v \notin B_m(R)$, $F'm'(v, t) = \widetilde{F}_{m'}(v, t)$ for every $t \in [0, 1]$.

This last configuration $(F'_m)_{m \in M}$ is the result of this application of R.

Notice that a given rule may admit several applications to a given configuration, yielding different results, depending on the objects chosen in the first

step. Notice moreover that the resulting objects have a non-zero similarity with the expected reactives that depends on the rule as well as on the input objects.

Now, as in the classical case, a *transition* for a membrane system Π consists of a maximal symultaneous application of rules: all steps (1) corresponding to rules being applied are perfomed symultaneously, then all steps (2) and finally all steps (3). The rules applied in a given transition are chosen in a non-deterministic way (or, in more involved models, in some regulated way), but so that for every m no further rule in $\mathcal{R}_m$ can be triggered simultaneously to them. In particular, a given rule can be triggered several times in the same transition, provided enough (inexact) copies of the catalysts and active reactives are available.

A finite sequence of transitions between configurations of a fuzzy P-system Π, starting with the initial configuration, is called a *computation* with respect to Π. A computation *halts* when it reaches a *halting configuration* where no rule can be triggered.

Given a halting computation C with halting configuration $(H(C)_m)_{m\in\overline{M}}$, the (crisp) multiset over I^+ *associated* to it is

$$\begin{array}{rccl} H_C: & I^+ & \to & \mathbb{N} \\ & t & \mapsto & \sum_{v\in V_{out}} H(C)_{m_{out}}(v,t) \end{array}$$

Thus, for every $t \in I^+$, $H_C(t)$ is the number of objects in the output region that, at the end of the computation, are copies of some output reactive with degree of exactitude t.

Then, the *output* of a halting computation C will be the fuzzy subset of $\mathbb{N}$

$$\begin{array}{rccl} Out_{\Pi,C}: & \mathbb{N} & \to & I \\ & n & \mapsto & \bigvee\{t \mid H_C(t) = n\} \end{array}$$

In words, $Out_{\Pi,C}(n)$ is the greatest degree of exactitude t in I for which, at the end of the computation C, there exist n objects in the output region that are copies of some output reactive with degree of exactitude t.

Finally, the fuzzy set of natural numbers *generated* by a fuzzy membrane system Π is the join of all the outputs of halting computations with respect to Π. This is the mapping $Gen_\Pi : \mathbb{N} \to I$ defined by

$$Gen_\Pi(n) = \bigvee_{C \text{ halting}} Out_{\Pi,C}(n), \qquad n \in \mathbb{N}.$$

Thus,

$$\begin{aligned} Gen_\Pi(n) &= \bigvee\Big\{\bigvee\{t \in I^+ \mid H_C(t) = n\} \mid C \text{ halting}\Big\} \\ &= \bigvee\{t \in I^+ \mid H_C(t) = n \text{ for some halting computation } C\}. \end{aligned}$$

Notice that, I being finite, this supremum is actually a maximum, and that if $H_C(t) \neq n$ for every halting computation C, then $Gen_\Pi(n) = \bigvee \emptyset = 0$.

Theorem 1 *A set of finite valued fuzzy natural numbers is r.e. if and only if it is generated by a fuzzy P system.*

The finite-valuedness of the fuzzy subsets of $\mathbb{N}$ generated by our fuzzy P-systems is due to the finiteness of the sets of rules and the initial configuration.

To end this paper, we would like to point out that, although formally correct, our specific approach has a drawback from the fuzzy mathematics point of view. The association to a multiset $H : I^+ \to \mathbb{N}$ of the fuzzy subset of $\mathbb{N}$

$$\begin{array}{rccl} \mathcal{C}(H): & \mathbb{N} & \to & I \\ & n & \mapsto & \bigvee\{t \mid H(t) = n\} \end{array}$$

that underlies our definition of the output of a halting computation with respect to a fuzzy P-system is not additive in any natural sense, and in particular it cannot be considered a fuzzy cardinality [2]. We have tried to use some specific simple fuzzy cardinalities in this step, and we have obtained that the resulting fuzzy P-systems did not generate all finite-valued recursively enumerable fuzzy subsets of $\mathbb{N}$, but we have not ruled out the possibility of using some other, cunningly chosen, fuzzy cardinality.

Our current research agenda includes this problem, as well as the problem of getting rid of the assumption used in this paper that an object can only be similar to one reactive.

References

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